Lesson 6: Finite and Infinite Decimals

Classwork

Opening Exercise

a. Use long division to determine the decimal expansion of $\frac{54}{20}$.

b. Use long division to determine the decimal expansion of $\frac{7}{8}$.

c. Use long division to determine the decimal expansion of $\frac{8}{9}$.

d. Use long division to determine the decimal expansion of $\frac{22}{7}$.

e. What do you notice about the decimal expansions of parts (a) and (b) compared to the decimal expansions of parts (c) and (d)?

Example 1

Consider the fraction $\frac{5}{8}$. Write an equivalent form of this fraction with a denominator that is a power of 10, and hence write the decimal expansion of this fraction.

Example 2

Consider the fraction $\frac{17}{125}$. Is it equal to a finite or an infinite decimal? How do you know?

Exercises 1–5

You may use a calculator, but show your steps for each problem.

1. Consider the fraction $\frac{3}{8}$.

   a. Write the denominator as a product of 2's and/or 5's. Explain why this way of rewriting the denominator helps to find the decimal representation of $\frac{3}{8}$.

   b. Find the decimal representation of $\frac{3}{8}$. Explain why your answer is reasonable.

2. Find the first four places of the decimal expansion of the fraction $\frac{43}{64}$.

3. Find the first four places of the decimal expansion of the fraction $\frac{29}{125}$.

4. Find the first four decimal places of the decimal expansion of the fraction $\frac{19}{34}$.

5. Identify the type of decimal expansion for each of the numbers in Exercises 1–4 as finite or infinite. Explain why their decimal expansion is such.

Example 3

Will the decimal expansion of $\frac{7}{80}$ be finite or infinite? If it is finite, find it.

Example 4

Will the decimal expansion of $\frac{3}{160}$ be finite or infinite? If it is finite, find it.

Exercises 6–8

You may use a calculator, but show your steps for each problem.

6. Convert the fraction $\frac{37}{40}$ to a decimal.

   a. Write the denominator as a product of 2's and/or 5's. Explain why this way of rewriting the denominator helps to find the decimal representation of $\frac{37}{40}$.

   b. Find the decimal representation of $\frac{37}{40}$. Explain why your answer is reasonable.

7. Convert the fraction $\frac{3}{250}$ to a decimal.

8. Convert the fraction $\frac{7}{1250}$ to a decimal.

Lesson Summary

Fractions with denominators that can be expressed as products of 2's and/or 5's are equivalent to fractions with denominators that are a power of 10. These are precisely the fractions with finite decimal expansions.

Example:

Does the fraction $\frac{1}{8}$ have a finite or an infinite decimal expansion?

Since $8 = 2^3$, then the fraction has a finite decimal expansion. The decimal expansion is found as

$$\frac{1}{8} = \frac{1}{2^3} = \frac{1 \times 5^3}{2^3 \times 5^3} = \frac{125}{10^3} = 0.125.$$

If the denominator of a (simplified) fraction cannot be expressed as a product of 2's and/or 5's, then the decimal expansion of the number will be infinite.

Problem Set

Convert each fraction given to a finite decimal, if possible. If the fraction cannot be written as a finite decimal, then state how you know. You may use a calculator, but show your steps for each problem.

1. $\frac{2}{32}$
2. $\frac{99}{125}$
3. $\frac{15}{128}$
4. $\frac{8}{15}$
5. $\frac{3}{28}$
6. $\frac{13}{400}$
7. $\frac{5}{64}$
8. $\frac{15}{35}$
9. $\frac{199}{250}$
10. $\frac{219}{625}$